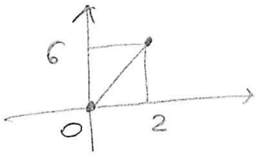
$\int$ curvilinei alcuni es. svolti

(cenno svolgimento)

M. Manfredini

1) $\begin{cases} x = t+2 \\ y = 3t+6 \end{cases}$ $\Leftrightarrow$ $y = 3x$

$t \in [-2, 0]$ $\qquad x \in [0, 2]$



$l(\gamma) = \sqrt{36+4} = 2\sqrt{10}$

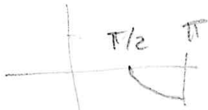
direttamente $\quad l(\gamma) = \int_{-2}^{0} \|\dot\gamma(t)\| dt = \int_{-2}^{0} \sqrt{10} = 2\sqrt{10}$.

2) $\dot\gamma(t) = (-4 \cos t \,\text{sen}\, t, \cos t \,\text{sen}\, t)$

$\dot\gamma(t) = 0 \Leftrightarrow \cos t \,\text{sen}\, t = 0 \Leftrightarrow t = \frac{\pi}{2}$ opp $t = \pi$

che corrispondono agli estremi della curva $\Rightarrow \gamma$ è regolare

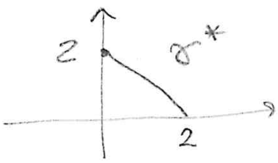
$l(\gamma) = \int_{\pi/2}^{\pi} 4\sqrt{2}\, \underbrace{|\cos t \,\text{sen}\, t|}_{\leq 0} dt = -4\sqrt{2} \int_{\pi/2}^{\pi} \cos t \,\text{sen}\, t\, dt$



$= 4\sqrt{2}\ \frac{1}{2}$

osserviamo che posto $\begin{cases} x = 2\cos^2 t \\ y = 2\,\text{sen}^2 t \end{cases}$ $\Rightarrow$ $x+y=2$

$x \in [0, 2]$



$y = 2-x$

3) $\dot\gamma(t) = 3(-\cos^2 t \,\text{sen}\, t, \text{sen}^2 t \cos t)$

$\dot\gamma(t) = 0 \Leftrightarrow t = 0, \frac{\pi}{2}, \frac{3}{2}\pi, 2\pi$

γ è unione di 4 curve regolari

$l(\gamma) = 4 \int_0^{\pi/2} 3\sqrt{\cos^4 t \,\text{sen}^2 t + \text{sen}^4 t \cos^2 t}\ dt =$

$\uparrow$ per la periodicità

$= 12 \int_0^{\pi/2} \sqrt{\cos^2 t \,\text{sen}^2 t}\ dt = 12 \int_0^{\pi/2} \cos t \,\text{sen}\, t = 6$

4) $\gamma \in C^1$

$$\dot\gamma(t) = \left(+\frac{1}{(1+t^2)^{3/2}}, \frac{-t}{(1+t^2)^{3/2}} \right) \qquad t \in [-1,1]$$

$$\|\dot\gamma(t)\| = \frac{1}{1+t^2} \neq 0$$

versore tangente $T(t) = \dfrac{\dot\gamma(t)}{\|\dot\gamma(t)\|} = \left(\dfrac{1}{\sqrt{1+t^2}}, \dfrac{-t}{\sqrt{1+t^2}} \right)$

parametro arco: per definizione $\quad s : [-1,1] \to [0, \ell(\gamma)]$

$$s(t) = \int_{-1}^{t} \|\dot\gamma(\tau)\| \, d\tau = \int_{-1}^{t} \frac{1}{1+\tau^2} d\tau = \operatorname{arctg} t + \frac{\pi}{4}$$

$$\ell(\gamma) = s(1) = \frac{\pi}{2}$$

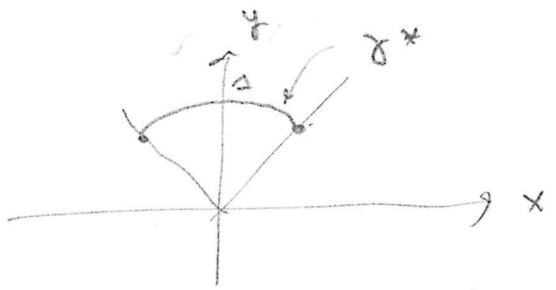
osserviamo che

$$\begin{cases} x = \dfrac{t}{\sqrt{1+t^2}} \\ y = \dfrac{1}{\sqrt{1+t^2}} \end{cases} \qquad x^2+y^2 = 1$$

$t=-1 \mapsto (x(-1), y(-1)) = \left(-\frac{1}{\sqrt2}, \frac{1}{\sqrt2}\right)$

$t=1 \mapsto (x(1), y(1)) = \left(\frac{1}{\sqrt2}, \frac{1}{\sqrt2}\right)$

Allora



5) $\gamma(t) = (t, t^2) \quad t \in [0,2] \qquad \dot\gamma(t) = (1, 2t) \qquad \|\dot\gamma(t)\| = \sqrt{1+4t^2}$

$$\ell(\gamma) = \int_0^2 \sqrt{1+4t^2}\, dt = \frac12 \int_0^{\operatorname{arcsenh}(4)} \cosh^2 s \, ds =$$

$$t = \frac12 \operatorname{senh} s$$

$$1 + \operatorname{senh}^2 s = \cosh^2 s \qquad \cosh^2 s = \frac{1+\cosh(2s)}{2}$$

$$= \frac12 \int_0^{\operatorname{arcsenh}(4)} \left(\frac12 + \frac{\cosh(2s)}{2} \right) ds =$$

$$= \frac14 \operatorname{arcsenh}(4) + \frac12 \left[\frac{\operatorname{senh}(2s)}{2} \right]_{s=0}^{\operatorname{arcsenh}(4)}$$

C.

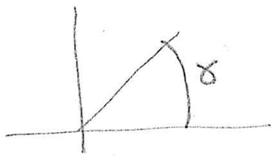
6) $\dot{\gamma}(t) = (e^t \cos t - e^t \operatorname{sen} t,\ e^t \operatorname{sen} t + e^t \cos t)$

$$\|\dot{\gamma}(t)\|^2 = e^{2t}\cos^2 t + e^{2t}\operatorname{sen}^2 t \cancel{- 2e^{2t} \operatorname{sen} t \cos t}$$
$$e^{2t}\operatorname{sen}^2 t + e^{2t}\cos^2 t \cancel{+ 2e^{2t} \operatorname{sen} t \cos t} =$$

$$= 2e^{2t}$$

$$\int_\gamma (x+1)\, ds = \int_0^\pi (e^t \cos t + 1)\, e^t = \dots$$

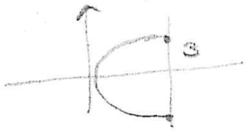
per parti

7) 

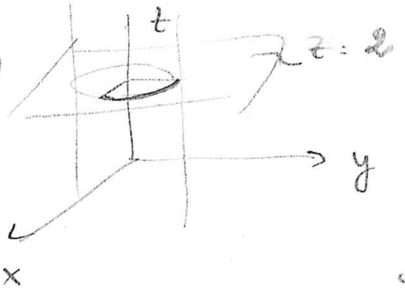
$\gamma(t) = (2\cos t, 2\operatorname{sen} t)$ $t \in [0, \pi/4]$

$$\int_\gamma xy\, ds = \int_0^{\pi/4} 4 \cos t \operatorname{sen} t\ 2\, dt =$$

8) $\int_\gamma y\, ds$ $\gamma(t) = (t^2, t)$ $t \in [-\sqrt{3}, \sqrt{3}]$



$$\int_\gamma y\, ds = \int_{-\sqrt{3}}^{\sqrt{3}} t\, ((2t)^2 + 1)^{1/2}\, dt = \text{—}$$

9) 

$\gamma(t) = (\sqrt{3}\cos t, \sqrt{3}\operatorname{sen} t, 2)$ $t \in [0, \pi/2]$

$$\|\dot{\gamma}(t)\|^2 = \sqrt{3}$$

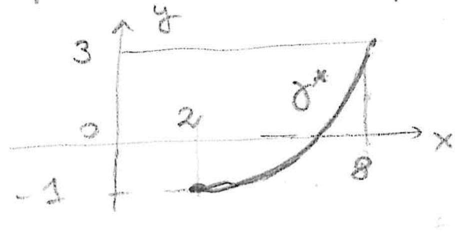
$$\int_\gamma x e^y z\, ds = \int_0^{\pi/2} 2\sqrt{3} \cos t\ e^{\sqrt{3}\operatorname{sen} t}\, dt = \dots$$

10) $\dot{\gamma}(t) = (3, 2t)$ $\|\dot{\gamma}(t)\|^2 = 9 + 4t^2$ $\gamma(t) = (2+3t, t^2-1)$ $t \in [0, 2]$

$$\int_\gamma (x-2)\, ds = \int_0^2 (2+3t-2)\,(9+t^2 4)^{1/2}\, dt = \dots$$

$\gamma(0) = (2, -1)$

$\begin{cases} x = 2+3t \\ y = t^2 - 1 \end{cases}$ $\begin{cases} t = (x-2)/3 \\ y = \dfrac{x^2+4-4x}{9} - 1 = \dfrac{x^2 - 4x - 5}{9} \end{cases}$ $\gamma(2) = (8, 3)$



$2x - 4 = 0$

$x = 2$

Vertice

$(2, -1)$

Ca

11) $\dot{\gamma}(t) = (1,\ 1+\log t) \neq (0,0) \quad t \in]1,2[$

$\|\dot{\gamma}(t)\| = \sqrt{1+(1+\log t)^2}$

$$\int_\gamma \frac{1}{x}\, ds = \int_1^2 \frac{1}{t}\sqrt{1+(1+\log t)^2}\, dt \underset{\substack{\uparrow \\ \sigma = 1+\log t}}{=} \int_1^{1+\log 2} \sqrt{1+\sigma^2}\, d\sigma = *$$

$\sigma = \operatorname{senh} u$

$u = \operatorname{senh}^{-1}(\sigma) = \log\left(\sigma + \sqrt{1+\sigma^2}\right) \qquad$ "$d\sigma = \cosh u\, du$"

$$* = \int_{\operatorname{senh}^{-1}(1)}^{\operatorname{senh}^{-1}(1+\log 2)} \cosh^2 u\, du = \frac{1}{2}\left(u + \operatorname{senh} u \cosh u\right)\Big]_{-}^{-}$$

lasciare indicato

12) $\dot{\gamma}(t) = (-\operatorname{sen} t,\ \cos t,\ 2t) \neq (0,0,0) \quad t \in]0,\pi[$

$$\int_\gamma \sqrt{z}\, ds = \int_0^\pi t\sqrt{1+4t^2}\, dt = \frac{1}{12}\left(1+4t^2\right)^{3/2}\Big]_0^\pi = \dots$$